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To H0 or not to H0?

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ABSTRACT
This paper investigates whether changes to late time physics can resolve the 'Hubble tension'. It is argued that many of the claims in the literature favouring such solutions are caused by a misunderstanding of how distance ladder measurements actually work and, in particular, by the inappropriate use of distance ladder H_0 priors. A dynamics-free inverse distance ladder shows that changes to late time physics are strongly constrained observationally and cannot resolve the discrepancy between the SH0ES data and the base ΛCDM cosmology inferred from *Planck*.

Key words: cosmology: cosmological parameters, large-scale structure of Universe, cosmic background radiation, observations

1 INTRODUCTION

As is well known, a six parameter ΛCDM cosmology[1] has proved to be spectacularly successful in explaining the cosmic microwave background radiation (CMB), light element abundances and a wide range of other astronomical data (e.g. Planck Collaboration et al. 2018; Efstathiou & Gratton 2019; Mossa et al. 2020; eBOSS Collaboration et al. 2020). As noted in Planck Collaboration et al. (2018), the agreement between the base ΛCDM model and observations is so good, that many researchers have begun to focus on possible discrepancies or 'tensions', with the hope that the model might break to reveal new truths about our Universe. This is reasonable given that many ingredients of the model, particularly the physics describing the dark sector, remain mysterious at this time.

The discrepancy between early time and late time determinations of the Hubble constant, H_0, is probably the most serious such tension. This tension became apparent following the first results from the *Planck* satellite (Planck Collaboration et al. 2014) which revealed a discrepancy between the best fit base ΛCDM value of H_0 and the Cepheid-based distance ladder measurement of H_0 by the SH0ES[2] collaboration (Riess et al. 2011). Since then, the 'Hubble tension' (as it has become known) has intensified: recent results from the SH0ES collaboration give $H_0 = 74.03 \pm 1.42$ km s^{-1}Mpc^{-1} (Riess et al. 2019, hereafter R19) (udpating the results of Riess et al. (2016), hereafter R16) which differs by 4.3σ from the base ΛCDM value $H_0 = 67.44 \pm 0.58$ km s^{-1}Mpc^{-1} inferred from the most recent analysis of *Planck* (Efstathiou & Gratton 2019). To add to the conundrum, the lower value of H_0 inferred from the CMB is in very good agreement with various applications of an inverse distance ladder, irrespective of whether the sound horizon, r_d, is fixed to a value determined from the CMB or to a value inferred from primordial nucleosynthesis (e.g. Aubourg et al. 2015; Verde et al. 2017; Addison et al. 2018; Abbott et al. 2018; Macaulay et al. 2019).

Possible modifications to the base ΛCDM model that might resolve this tension have been discussed in the reviews by Knox & Millea (2020), Beenakker & Venhoek (2021) and Di Valentino et al. (2021). In broad brush, the proposed solutions fall into four categories: (i) radical departures from conventional cosmology, including departures from General Relativity[3]; (ii) changes to the physics of the early Universe (for example adding additional relativistic species, or neutrino interactions); (iii) new physics at matter-radiation equality, or recombination, that alters the value of the sound horizon, (iv) changes to the physics at late times. The focus of this paper is on solutions in class (iv).

The issue of whether the tension is real is not yet fully clear (see Freedman et al. 2019; Yuan et al. 2019; Freedman et al. 2020). Despite the fact that the author is an unashamed tension skeptic (Efstathiou 2020), I will take the SH0ES results at face value in this paper and consider whether the Hubble tension can be resolved by modifications to late time physics.

Figure 1 shows various measurements of $H(z)$ from baryon acoustic oscillation (BAO) experiments. The normalization of these measurements assumes the *Planck* value of the sound horizon

$$r_d = 147.31 \pm 0.31 \text{ Mpc}. \quad (1)$$

Throughout this paper we will assume that the base ΛCDM

[1] Which I will refer to as the base ΛCDM model.
[2] SNe, H_0, for the Equation of State of dark energy
[3] Such models will not be considered further in this paper.

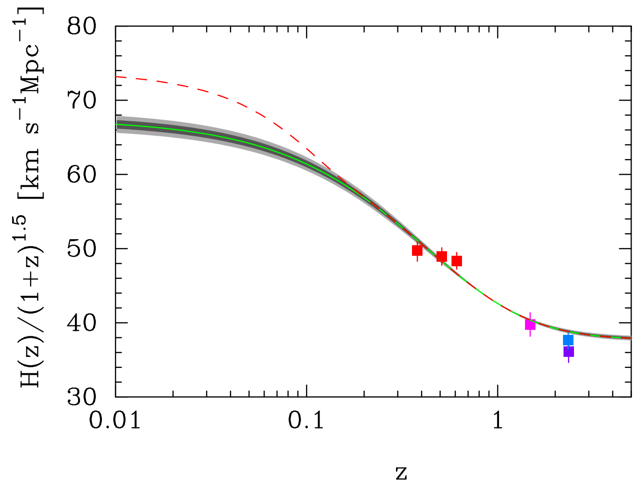

Figure 1. The evolution of the Hubble parameter with redshift. The red points[5] show $H(z)$ measurements in three redshift bins inferred from galaxy correlations in the Baryon Oscillation Spectroscopic Survey (BOSS) Alam et al. (2017). The purple point at $z = 2.35$ shows $H(z)$ from BAO features in the cross-correlations of Lyα absorbers and quasars (Blomqvist et al. 2019). The blue point at $z = 2.34$ shows $H(z)$ from BAO features in the correlations of Lyα absorbers (de Sainte Agathe et al. 2019). The magenta point at $z = 1.48$ shows $H(z)$ from BAO feaures in the correlations of quasars (Hou et al. 2020). The green line shows $H(z)$ for the best-fit base ΛCDM determined from *Planck* and the grey bands show 1σ and 2σ ranges. The dashed line shows Eq. (2) with parameters chosen to match the SH0ES value of H_0 at $z = 0$.

model describes accurately the physics at early times and so r_d is fixed to Eq. (1). The sources for the observational data points are listed in the figure caption. The green line shows $H(z)$ for the best fit base ΛCDM cosmology determined from *Planck* and the grey bands show 1σ and 2σ ranges. The green line approaches the value $H_0^P = 67.44$ km s^{-1}Mpc^{-1} asymptotically as $z \to 0$. As long as r_d remains fixed, apparently the only way to reconcile the BAO data with the SH0ES value of H_0 is to modify the base ΛCDM curve. For example, the dashed line in Fig. 1 shows the relation

$$H(z) = H_0^f\Big[\Omega_m(1+z)^3 + (1-\Omega_m)\left(1+\Delta\exp(-(z/z_c)^\beta)\right)\Big]^{1/2}, \qquad (2)$$

with parameters $H_0^f = H_0^P$, $\Omega_m = 0.31$, $\Delta = 0.30$, $z_c = 0.1$ and $\beta = 2$. With this choice of parameters, the value of H_0 matches the SH0ES value whilst matching the BAO $H(z)$ measurements at $z > 0.3$.

If the dashed curve is interpreted as a variation in the equation of state of the dark energy, then it necessarily requires a phantom equation of state, $w < -1$, at low redshifts. Alternatively, one might imagine that transference of energy between the dark matter and dark energy results in something like the dashed curve. Models of both types have been proposed as 'solutions' to the Hubble tension as summarized in Di Valentino et al. (2021). *These 'solutions' are not viable because the SH0ES team does not directly measure H_0.*

In fact, the SH0ES team measure the absolute peak magnitude, M_B, of Type Ia supernovae (SN) by calibrating the distances of SN host galaxies to local geometric distance anchors via the Cepheid period luminosity relation. The magnitude M_B is then converted into a value of H_0 via the magnitude-redshift relation of the Pantheon SN sample (Scolnic et al. 2017) of supernovae in the redshift range $0.023 < z < 0.15$. All of the proposed late time 'solutions' to the Hubble tension reviewed in Di Valentino et al. (2021) interpret the SH0ES H_0 measurement as a measurement of the value of $H(z)$ as $z \to 0$ (often imposing a SH0ES 'H_0 prior') without investigating whether the 'solution' is consistent with the magnitude-redshift relation of Type Ia SN. It is hardly advancing our understanding if authors propose solutions to the H_0 tension that are inconsistent with the measurements that they are trying to explain.

This point has been made previously by Lemos et al. (2019), Benevento et al. (2020) and most recently by Camarena & Marra (2021), but has been comprehensively ignored in recent literature. The purpose of this paper is to show how theoretical models exploring new physics at late time should be compared with distance ladder measurements. I will adopt a 'dynamics free' approach to this problem and show that late time modifications of the ΛCDM cosmology cannot resolve the Hubble tension.

2 THE INVERSE DISTANCE LADDER

We will write the metric of space-time as

$$ds^2 = c^2dt^2 - R^2(t)(dx^2 + dy^2 + dz^2), \qquad (3)$$

adopting a spatially flat geometry consistent with the very tight experimental constraints on spatial curvature (Efstathiou & Gratton 2020). The Hubble parameter, $H = R^{-1}dR/dt$, then fixes the luminosity distance $D_L(z)$ and comoving angular diameter distance $D_M(z)$ according to

$$D_L(z) = c(1+z)\int_0^z \frac{dz'}{H(z')}, \quad D_M(z) = \frac{D_L(z)}{(1+z)}. \qquad (4)$$

Standard candles and standard rulers can therefore be used to constrain $H(z)$ independently of any dynamics (Heavens et al. 2014; Bernal et al. 2016; Lemos et al. 2019; Aylor et al. 2019) . As long as the relations of Eq. (4) are satisfied, it does not matter whether modifications to the functional form of $H(z)$ are caused by by changes to the equation of state of dark energy or interactions between dark matter and dark energy.

A standard candle with absolute magnitude M at redshift z will have an apparent magnitude

$$m = M + 25 + 5\log_{10} D_L(z), \qquad (5a)$$
$$= -5a + 5\log_{10} c\hat{d}_L(z) \qquad (5b)$$

with D_L in units of Mpc. In (5b), a is the intercept of the magnitude-redshift relation, $5a = -(M + 25 - 5\log_{10} H_0)$ and $\hat{d}_L(z) = H_0 D_L(z)/c$. The SH0ES Cepheid data allow one to calibrate the absolute magnitude M_B of Type Ia SN.

[5] The data and covariance matrices are from the file BAO_consensus_covtot_dM_Hz.txt downloaded from http://www.sdss3.org/science/BOSS_publications.php

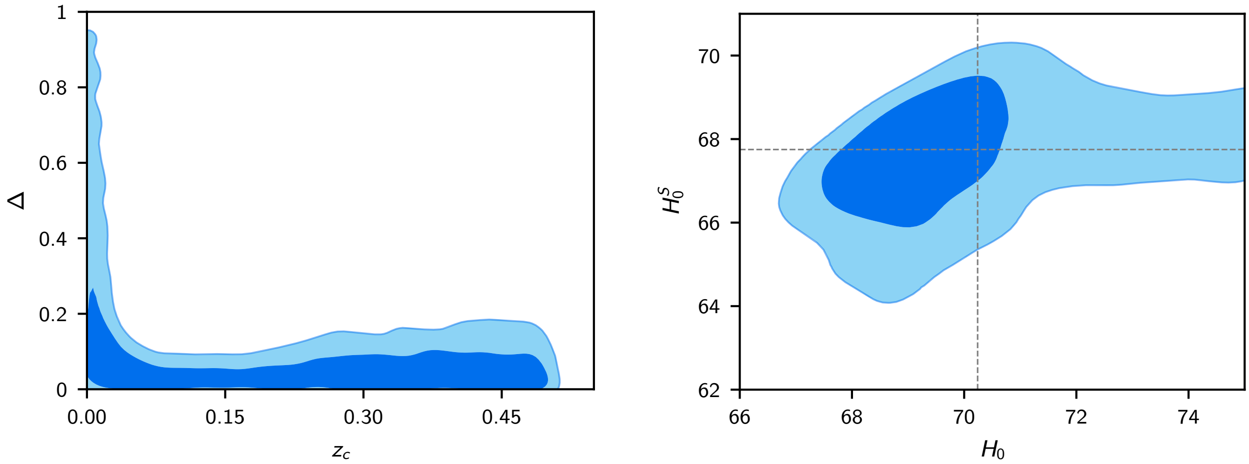

Figure 2. 68 and 95% constraints on the parameters Δ and z_c (left hand panel) and the SH0ES-like parameter H_0^S of Eq. (10b) and H_0. (right hand panel).

Combining the geometrical distance estimates of the maser galaxy NGC 4258 (Reid et al. 2019), detached eclipsing binaries in the Large Magellanic Cloud (Pietrzyński et al. 2019) and parallax measurements for 20 Milky Way Cepheids (Benedict et al. 2007; van Leeuwen et al. 2007; Riess et al. 2018), the SH0ES Cepheid photometry and Pantheon SN peak magnitudes, I find

$$M_B = -19.244 \pm\ 0.042 \text{ mag}. \tag{6}$$

To estimate H_0, R16 determine the intercept of the Pantheon SN magnitude-redshift relation by fitting the low redshift expansion to the luminosity distance

$$\hat{d}_l(z) = z\left[1 + (1-q_0)\frac{z}{2} - \frac{1}{6}(1 - q_0 - 3q_0^2 + j_0)z^2\right], \tag{7}$$

over the redshift range $z = 0.023$ to $z = 0.15$, with the deceleration and jerk parameters set to $q_0 = -0.55$ and $j_0 = 1$ (close to the values for base ΛCDM $q_0 = -0.535$, $j_0 = 1$). They find

$$a_B = 0.71273 \pm 0.00176. \tag{8}$$

Together with Eq. (7), this gives

$$H_0 = 73.1 \pm 1.4 \text{ km s}^{-1}\text{Mpc}^{-1}, \tag{9}$$

slightly lower than the value of H_0 quoted by R19 (reflecting differences in the period ranges and photometric samples for the LMC and M31 used in my analysis compared to those used by R19). These differences are unimportant for this paper. If one has reason to prefer the R19 value, then one can adopt a value of M_B that is fainter than Eq. (7) by 0.027 magnitudes.

To apply the inverse distance ladder, I follow closely the analysis described in Lemos et al. (2019). $H(z)$ is parameterized by Eq. (2) and the parameters of the model are determined by fitting to Pantheon SN magnitudes and the BAO $D_M(z)$ and $H(z)$ measurements from the references given in the caption to Fig. 1, supplemented by the $D_V(z) = (D_M^2(z)cz/H(z))^{1/3}$ measurement at $z = 0.106$ from Beutler et al. (2011). The free parameters of the model are H_0^f, Ω_m, Δ, z_c, β and M_B with uniform priors as listed in Table 1. To compare with the BAO results, I adopt a Gaussian prior on the sound horizon r_d with the parameters of Eq. (1). I use the `MULTINEST` sampling algorithm (Feroz et al. 2009, 2011) to explore the parameter space.

Table 1. Results of applying the inverse distance ladder. The table lists the mean values of the parameters and their 1σ error. The last column lists the ranges over which a uniform prior is applied to the parameters. The parameters H_0 and H_0^S are derived parameters (see Eqs. 2 and 10b). The units of H_0^f, H_0 and H_0^S are km s^{-1}Mpc^{-1}.

parameter	fit	prior range
H_0^f	68.13 ± 1.00	$60 - 80$
Ω_m	0.306 ± 0.017	$0.25 - 0.35$
Δ	0.107 ± 0.162	$0.0 - 1.0$
z_c	0.167 ± 0.091	$0.001 - 0.5$
β	2.45 ± 0.86	$1.0 - 4.0$
M_B	-19.387 ± 0.021	$-19.0 - -19.5$
H_0	70.5 ± 3.6	–
H_0^S	67.73 ± 0.97	–

The constraints on these parameters are summarized in Table 1. The left hand plot in Fig. 2 shows the 1σ and 2σ constraints on the parameters Δ and z_c[6]. The key point here is that the parameter Δ, and therefore $H(z)$ is well constrained for values of $z_c \gtrsim 0.05$ because at these redshifts the Hubble parameter is tightly constrained by the SN magnitude-redshift relation. At lower values of z_c, the parameter Δ becomes poorly constrained by the SN magnitude-redshift relation and solutions with high values of H_0 are allowed. This reinforces the conclusions of Benevento et al. (2020) and Camarena & Marra (2021) that the SN data are insensitive to late time changes in the dark energy equation of state[7].

[6] Evidently, the parameter z_c runs into the upper range of its prior, but this is unimportant.
[7] It is worth mentioning that the SN host galaxies of R16 and

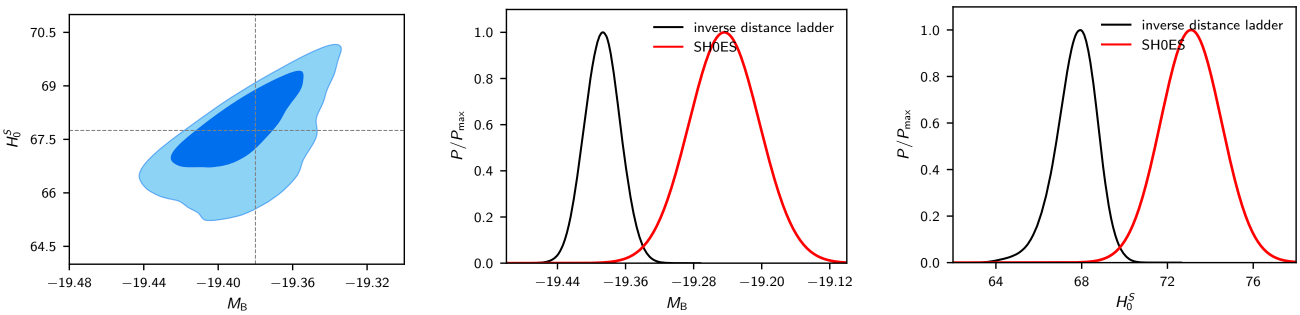

Figure 3. The left hand panel shows 68 and 95% constraints on the parameters H_0^S and M_B. The dotted lines show the mean values of these parameters listed in Table 1. The midle panel shows the marginalised posterior distributions of the SN peak absolute magnitude M_B determined from the inverse distance ladder discussed in this paper (black line) compared with the posterior distribution of M_B determined from the SH0ES data (red line). The right hand panel shows the equivalent plot, but for the parameter H_0^S instead of M_B.

We can compute a derived quantity H_0^S that is equivalent to the SH0ES estimate of H_0

$$a_B = \left(\sum_{ij} C_{ij}^{-1}(\log_{10}\hat{d}_L(z) - 0.2m_B(i))\right) / \sum_{ij} C_{ij}^{-1} \qquad (10a)$$

$$H_0^S = 10^{0.2(M_B+5a_B+25)} \qquad (10b)$$

where C is the covariance matrix of the Pantheon SN magnitudes and the sums in Eq. (10a) extend all SN in the Pantheon sample with redshifts in the range $0.023 - 0.15$.

The right hand plot in Fig. 2 shows H_0^S plotted against the true value of H_0. One can see the long tail extending to high values of H_0. These high values arise in solutions with $z_c \lesssim 0.05$ and high values of Δ corresponding to phantom-like equations of state. However, the SH0ES analysis is oblivious to these high values of H_0. Instead, a SH0ES type analysis would infer a value close to the estimate H_0^S of Eq. (10b) which is always low. For these models $H_0^S = 67.75 \pm 1.01$ km s^{-1}Mpc^{-1}, discrepant with Eq. (9) by[8] 3.1σ *despite the ability of the model to mimic extreme phantom-like equations-of-state.* It is also worth noting that the parameters Ω_m and H_0^S are each within about 0.4σ of the base ΛCDM values inferred from *Planck*. There is not even a hint from these data for any phantom-like physics.

The discrepancy between the inverse distance ladder and the SH0ES data is illustrated clearly in Fig. 3. The left hand panel shows the 1σ and 2σ constraints on H_0^S and M_B. The central panel shows the posterior distribution of M_B determined from the inverse distance ladder (black line) compared with the result of Eq. (7) derived from the SH0ES data (red line). The right hand panel shows the posterior distribution of H_0^S compared with Eq. (9). The SH0ES results are clearly discrepant with the inverse distance ladder. As long as the *Planck* value of r_d is correct and the relations of Eq. (4) apply, *modifications to late-time physics cannot explain the SH0ES data.*

Freedman et al. (2019) are very nearby, with redshifts $z \lesssim 0.007$. Yet as shown in Fig. 7 of Freedman et al. (2019), their velocity flow corrected distances define a Hubble diagram with very little scatter. There is therefore no evidence for an abrupt change to the equation of state at very low redshifts.

[8] Or 3.8σ if we compare with the R19 analysis.

3 CONCLUSIONS AND DISCUSSION

Authors of numerous papers investigating late time solutions to the Hubble tension seem to be unaware of how distance ladder measurements actually work. Even worse, some authors impose the SH0ES H_0 value as a prior on the Hubble parameter at $z = 0$ leading to claims of evidence for phantom dark energy, dark matter-dark energy interactions, or other exotic late-time physics. The review article by Di Valentino et al. (2021) cites many such examples.

If one wants to investigate consequences of new late-time physics, the simplest way to compare with the SH0ES results is to drop H_0 as a parameter in favour of the SN peak absolute magnitude M_B, i.e. rather than explaining the 'Hubble tension' one should instead focus on the 'supernova absolute magnitude tension'. The goal then is to find a late time solution that brings M_B into agreement with the SH0ES measurement. This necessarily involves analysing the Pantheon SN sample[9]. If one wants to combine the SH0ES data with other astrophysical data to constrain late time physics, then one should impose a SH0ES prior on the parameter M_B and not on the parameter H_0. An alternative approach is to use the SH0ES-like parameter H_0^S in place of M_B.

However, using the Pantheon and BAO data, the inverse distance ladder places very strong constraints on new physics at late times. The results of Table 1 show that the data are in excellent agreement with the base ΛCDM cosmology determined from *Planck*. BAO is now a mature field employing analysis techniques that have been tested extensively against simulations. There is no good reason to ignore these measurements. Neither is there a good reason to ignore the Pantheon SN sample, since this is an essential part of the SH0ES distance ladder. It is, therefore, unlikely that changes to late time physics can resolve the 'Hubble tension'. This conclusion is independent of any dynamics, and independent of perturbations insofar as the *Planck* value of r_d is unaltered.

[9] Similar remarks apply to the tip of the red giant branch distance ladder Freedman et al. (2019), but with the Carnegie Supernova Project (Hamuy et al. 2006; Krisciunas et al. 2017) replacing the Pantheon sample.

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